Assignment 10

This homework is due Friday April 15.

There are total 30 points in this assignment. 27 points is considered 100%. If you go over 27 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 6.4–6.5 of Textbook.

(1) [3pt] Determine the domain of analyticity for the following functions and evaluate $\int_{C_1^+(0)} f(z)dz$ using Cauchy–Goursat theorem (or Extended Cauchy–Goursat theorem).
   (a) $f(z) = \frac{1}{4z^2-4z+5}$,
   (b) $f(z) = \frac{1}{4z^2-4z+3}$.

(2) [5pt] Evaluate $\int\limits_C \frac{dz}{(2z-1)(2z+3)}$ using Cauchy–Goursat theorem (or Deformation of contour/Extended Cauchy–Goursat theorem if appropriate) for
   (a) the circle $C = C_{1/4}^+(0)$.
   (b) the circle $C = C_1^+(0)$.
   (c) the circle $C = C_3^+(0)$.

(3) [12pt] Evaluate the following integrals by using fundamental theorem (Definite integrals theorem). In each case explain why the use of the theorem is justified. In each case determine if using a different contour with the same endpoints can change the answer.
   (a) $\int_C z^2\,dz$, where C is the line segment from $1+i$ to $2+2i$.
   (b) $\int_C \sin z\,dz$, where C is the line segment from $-i$ to $-1+i$.
   (c) $\int_C ze^z\,dz$, where C is the line segment from $-1+i\frac{\pi}{2}$ to $2-i\pi$.
   (d) $\int_C \frac{1+z}{z}\,dz$, where C is the line segment from 1 to $-i$.
   (e) $\int_C \cos\frac{z}{3}\,dz$, where C is the line segment from 0 to $\pi+3i$.
   (f) $\int_C \sin^2 z\,dz$, where C is the line segment from 0 to $-i$.
   (g) $\int_C \frac{dz}{z^2-z}$, where C is the line segment from 2 to $2-i$.

(4) [10pt] Use Cauchy Integral formula (or that for derivatives, where appropriate) to find the following integrals.
   (a) $\int_{C_{20}^+(15i)} \frac{e^{2z}+\cos z}{z}dz$.
   (b) $\int_{C_1^+(1)} \frac{zdz}{(z+1)(z-1)^2}$.
   (c) $\int_{C_3^+(0)} \frac{\sin 5z}{z^4}dz$.
   (d) $\int_{C_{1/2}^+(0)} \frac{e^z dz}{z(z-1)}$.
   (e) $\int_{C_2^+(0)} \frac{e^{2z}dz}{z(z-1)}$. (*Hint:* Switch to integrals along $C_\varepsilon(0)$ and $C_\varepsilon(1)$ by Extended Cauchy–Goursat Theorem. At 0, think $\frac{e^{2z}/(z-1)}{z}$. At 1, think $\frac{e^{2z}/z}{z-1}$.)
   (f) $\int_{C_1^+(0)} \frac{dz}{z\cos z}$.